General Physics

A CHAOTICALLY DRIVEN IMPACT OSCILLATOR

Adam D. Hall , Tyson R. Shepherd, Brian K. Clark *, Epaminondas Rosa, Jr., Campus Box 4560, Department of Physics, Illinois State University, Normal, IL 617690-4560. bkc@phy.ilstu.edu

An impact oscillator is essentially a system driven with a periodic forcing function, such as a ball bouncing on a table driven by a sinusoidally varying voltage. The impact oscillator has been used to model cosmic rays interacting with magnetic fields [1], and may be an appropriate model of biological systems like stereocilia, for example. In each of these examples a mechanical system responds nonlinearly to a periodic driving force. In this work, we employ an electronic circuit that appears very much like a ball bouncing on an oscillating table as the impact oscillator [2]. This system can exhibit periodic and chaotic trajectories when a periodic driving source is employed. We examine the trajectories when the driving source is a chaotic oscillation obtained from the Chua circuit [3]. Trajectories of the electronic ball are recorded on computer for analysis.

The correlation dimension calculated for a series of embedding dimensions provides one estimate of the periodic (integer dimension), chaotic (fractional dimension), or random (dimension proportional to embedding dimension) nature of a trajectory. Correlation dimensions for representative experimental and simulated trajectories are calculated following the prescription of Grassberger and Procaccia [4]. The position time series recorded every millisecond is used to form a reconstructed phase space of embedding dimension, N, according to

$$\mathbf{X}_i = \{x(t_i), x(t_i+T), ..., x(t_i+(N-1)T)\}, \qquad (1)$$

where the maximum value of i is $n = 10000$ and T is the delay time between components in this work. The first components of adjacent vectors are selected so that $t_{i+1} - t_i = 2\,T$. The correlation sum for a given embedding dimension and set of vectors is calculated as

$$C(N,r) = \left(\frac{2}{n^2}\right)\sum_{i=1}^{n-5}\sum_{j=i+5}^{n}\Theta\left(r - \left|X_i - X_j\right|\right), \qquad (2)$$

where Θ is the step function, i.e. $\Theta(y) = 1$ if $y > 0$ and $\Theta(y) = 0$ otherwise. The correlation dimension is taken as the slope of log $C(N,r)$ versus log r when a stable value independent of N is achieved.

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